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# ON THE NOISE OF A TRANSISTOR WITH D.C. CURRENT CROWDING

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**Abstract**

Due to the ohmic potential drop in the active base region of a transistor d.c. current crowding may occur. The purpose of this paper is to examine the influence of this d.c. current crowding on the noise properties of the transistor. If the active base region is represented by a resistance $r_{bb'}$ in an equivalent circuit, the effective noise temperature of this base resistance decreases with increasing d.c. current crowding. At the same time the shot-noise current sources over the emitter–base and base–collector junctions terminate at a point of the base resistance $r_{bb'}$. This point divides the base resistance in a part $p\, r_{bb'}$ at the internal-base-point side and a part $(1-p)\, r_{bb'}$ at the external-base-point side. With increasing d.c. current crowding the value of $p$ increases.

## List of symbols

| | |
|---|---|
| $x$ | lateral position in the base |
| $W$ | half width of the base region (fig. 1) |
| $I_E$, $I_B$, $I_C$ | external emitter, base, collector d.c. currents |
| $i_E$, $i_B$, $i_C$ | external emitter, base, collector a.c. currents |
| $V_e$ | applied emitter d.c. voltage (base grounded) |
| $v_e$ | applied emitter a.c. voltage (base grounded) |
| $J_e(x)$ | emitter current density |
| $J_s$ | reverse saturation emitter current density of emitter–base junction |
| $I_b(x)$ | d.c. base current flowing in $x$-direction (majority carriers) |
| $i_b(x)$ | a.c. base current flowing in $x$-direction (majority carriers) |
| $V(x)$ | d.c. potential in the base |
| $v(x)$ | a.c. potential in the base |
| $i_n(x)$ | shot-noise current source in the base per unit length |
| $e_n(x)$ | thermal-noise voltage source in the base per unit length |
| $\varrho_{sh}$ | sheet resistance of the base region |
| $\Delta f$ | effective bandwidth of noise sources |
| $q$ | elementary charge |
| $k$ | Boltzmann's constant |
| $T$ | ambient temperature in °K |
| $t_{eff}$ | effective noise temperature divided by $T$ |
| $\alpha$ | current gain of the transistor |
| $r_{bb'}$ | base resistance of the transistor |
| $r_e$ | emitter differential resistance of the transistor |

## 1. Introduction

The lumped-model small-signal equivalent circuits of transistors have included a base resistance $r_{bb'}$ from the internal base point b' to the external base point b. A simple example of such an equivalent circuit in common base configuration is given in fig. 3. A part of the base resistance represents the contribution of the active base region under the emitter, the rest includes the resistance of the inactive base region and the base contact resistance. The noise contribution of $r_{bb'}$ in the white-noise region is commonly supposed to be given by the thermal-noise voltage[1,2]. This will be true for that part of the base resistance which is caused by the inactive base region. It is a series resistance from the external base contact to the active region of the base. The part of the base resistance caused by the active base region however is an equivalent resistance calculated with a distributed model of the transistor. To calculate the noise in such a transistor we have to add the noise courses to each elementary part of the distributed model and integrate over the whole active base region. Especially when current crowding is occurring due to the potential drop in the base region it is not justified to attribute a thermal-noise voltage source simply to the equivalent resistance of the active base region. Patterson [3]) has calculated the influence of a.c. current crowding in the base on the noise contribution of the base resistance. He assumed that no d.c. current crowding occurred. This assumption simplifies the calculations, for then each elementary transistor has equal small-signal parameters and noise sources.

It is the purpose of this paper to examine the noise behaviour of the transistor in which d.c. current crowding occurs. This case is important, for there may be an appreciable d.c. current crowding in modern high-frequency transistors with very thin base regions, or in transistors used in integrated circuits with wide emitters.

The structure for which we calculated the noise is given in fig. 1. It essentially

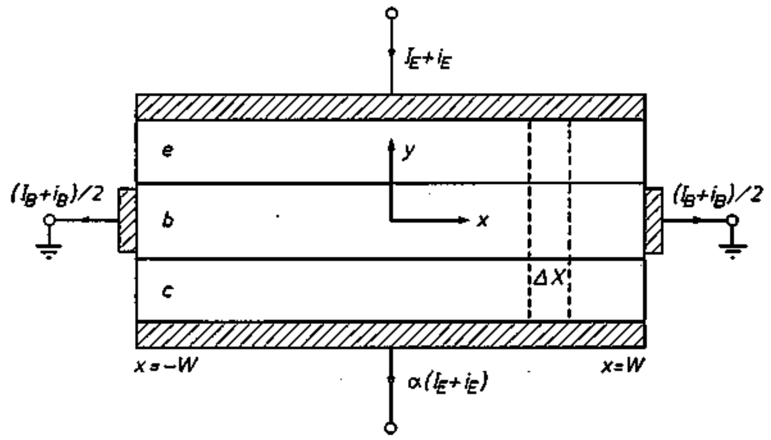


Fig. 1. Cross-section of the model used for the transistor, with external flowing currents.

resembles the model used by Hauser for calculation of the d.c. current crowding[4]). The base contact is assumed to be directly connected to the active base region. The base resistance of the inactive part of the base region of real transistors can always be added later, together with its thermal-noise voltage source.

In the distributed model the minority-carrier current in the base, injected from the emitter, is assumed to flow in the $y$-direction (fig. 1). The majority-carrier current in the base flows in the $x$-direction and causes a lateral potential drop in the base region.

## 2. Equations determining the base behaviour

We consider a small part $\Delta x$ of the base region. The base is assumed to extend over a unit length in the direction perpendicular to the $x$–$y$ plane.

The current density flowing in from the emitter is given by

$$J_e(x) = J_s \left\{ \exp \left[ \frac{q}{kT} (V_e - V(x)) \right] - 1 \right\}. \tag{1}$$

A part $\alpha$ of this current density flows into the collector. The part $(1 - \alpha)$ of the emitter current density has to be supplied from the base contacts and is determined by the emitter efficiency and the recombination in the base region. We assume $\alpha$ to be independent of position and current density. This is no essential limitation; every given dependence of $\alpha$ on the current density can be included in the calculations.

The total current flowing into the region $\Delta x$ has to be zero, and that condition gives us the following equation for $\Delta x \to 0$ (fig. 2):

$$\frac{\mathrm{d}I_b(x)}{\mathrm{d}x} = (1 - \alpha)\, J_s \left\{ \exp \left[ \frac{q}{kT} (V_e - V(x)) \right] - 1 \right\}. \tag{2}$$

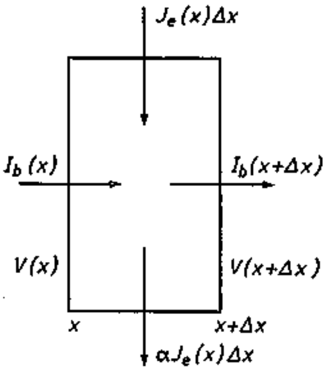


Fig. 2. Elementary part $\Delta x$ of the base region.

The base current flowing in the lateral direction of the base causes an ohmic potential drop determined by

$$\frac{dV(x)}{dx} = -\varrho_{sh}\, I_b(x). \tag{3}$$

Combining eqs (2) and (3) we get

$$\frac{d^2V}{dx^2} + \varrho_{sh}\,(1-\alpha)\, J_s \left\{\exp\left[\frac{q}{kT}\,(V_e - V(x))\right] - 1\right\} = 0. \tag{4}$$

If the base contacts are grounded, the boundary conditions for this second-order non-linear differential equation are

$$V(W) = V(-W) = 0. \tag{5}$$

In order to get the equations determining the small-signal a.c. behaviour we superpose small-signal a.c. quantities to the respective d.c. quantities in eqs (2), (3) and (4). By taking only the first-order disturbance in the d.c. equations we get the linear a.c. equations. To these equations we add the noise sources, and after introducing the function

$$F(x) = \varrho_{sh}\,(1-\alpha)\, J_s \frac{q}{kT} \exp\left[\frac{q}{kT}\,(V_e - V(x))\right] \tag{6}$$

we get:

$$\frac{di_b(x)}{dx} = \frac{F(x)}{\varrho_{sh}}\,(v_e - v(x)) + i_n(x), \tag{7}$$

$$\frac{dv(x)}{dx} = -\varrho_{sh}\, i_b(x) + e_n(x), \tag{8}$$

$$\frac{d^2v(x)}{dx^2} - F(x)\, v(x) = -F(x)\, v_e - \varrho_{sh}\, i_n(x) + \frac{de_n(x)}{dx}, \tag{9}$$

with boundary conditions

$$v(W) = v(-W) = 0. \tag{10}$$

The shot-noise term which is added in eq. (7) is given by

$$\overline{i_n(x)\, i_n^*(x')} = 2q\,(1-\alpha)\, J_e(x)\, \delta(x-x')\, \Delta f, \tag{11}$$

where $\delta$ is the delta function.

The thermal-noise source $e_n(x)$ of eq. (8) is given by

$$\overline{e_n(x)\, e_n^*(x')} = 4\, kT\, \varrho_{sh}\, \delta(x - x')\, \Delta f. \qquad (12)$$

The noise sources $i_n(x)$ and $e_n(x)$ are uncorrelated, for they are determined by two independent physical processes:

$$\overline{i_n(x)\, e_n^*(x')} = 0. \qquad (13)$$

For the linear second-order differential eq. (9) we introduce a Green's function determined by

$$\frac{d^2 G(x,x')}{dx^2} - F(x)\, G(x,x') = \delta(x - x'), \qquad (14)$$

and with boundary conditions

$$G(W,x') = G(-W,x') = 0; \qquad (15)$$

$G(x,x')$ has the elementary property of being symmetrical in $x$ and $x'$ [5]):

$$G(x,x') = G(x',x). \qquad (16)$$

### 3. Calculation of the base resistance

For a certain applied emitter voltage $V_e$, the potential in the base $V(x)$ is determined by the differential equation (4). If this equation is solved the function $F(x)$ (6) is also known.

The total d.c. base current flowing is then determined by

$$I_B = \int_{-W}^{W} \frac{dI_b(x)}{dx}\, dx = (1 - \alpha)\, J_s \int_{-W}^{W} \left\{ \exp\left[ \frac{q}{kT} (V_e - V(x)) \right] - 1 \right\} dx. \qquad (17)$$

For the calculation of the small-signal quantities we apply an emitter voltage $v_e$ and ignore the noise contributions. The voltage $v(x)$ in the base is then determined by eq. (9) without the noise terms, and we immediately see that $v(x)$ is linearly dependent on $v_e$. We therefore solve eq. (9) for $v_e = 1$ and call the solution $v_s(x)$. With the help of the Green's function we may also write:

$$v_s(x) = - \int_{-W}^{W} F(x')\, G(x,x')\, dx'. \qquad (18)$$

The total a.c. base current flowing is given, with the help of eq. (7), by

$$i_B = \int_{-W}^{W} \frac{di_b}{dx}\,dx = v_e \int_{-W}^{W} \frac{F(x)}{\varrho_{sh}} (1 - v_s(x))\,dx. \tag{19}$$

We replace the transistor by the equivalent circuit given in fig. 3 with an

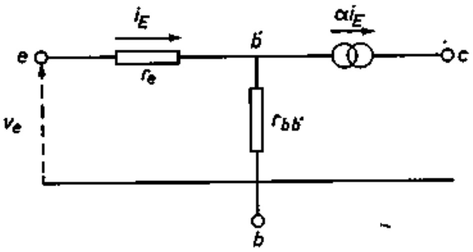


Fig. 3. Small-signal equivalent circuit of the transistor.

emitter differential resistance $r_e = kT/qI_E$ and base resistance $r_{bb'}$. An applied emitter a.c. voltage gives a base current $i_B$ determined by

$$v_e = i_E \frac{kT}{q\,I_E} + i_B\, r_{bb'} = i_B \left(\frac{kT}{q\,I_B} + r_{bb'}\right). \tag{20}$$

The equivalent circuit represents the transistor if eqs (19) and (20) are equivalent. This condition gives an equation for the $r_{bb'}$:

$$r_{bb'} = \left\{\int_{-W}^{W} \frac{F(x)}{\varrho_{sh}} (1 - v_s(x))\,dx\right\}^{-1} - \frac{kT}{q\,I_B}. \tag{21}$$

## 4. Calculation of the noise of the distributed transistor

We want to calculate the mean square $\overline{i_{Bn}{}^2}$ of the base noise current when the input e–b is short-circuited ($v_e = 0$).

We call the noise voltage in the base $v_n(x)$, which is determined by eq. (9), with $v_e = 0$.

With the help of the Green's function we can write:

$$v_n(x) = \int_{-W}^{W} G(x,x') \left\{-\varrho_{sh}\, i_n(x') + \frac{de_n(x')}{dx'}\right\} dx'. \tag{22}$$

The noise current $i_{Bn}$ is then obtained by integration of eq. (7) with $v_c = 0$:

$$i_{Bn} = \int_{-W}^{W} \frac{\mathrm{d}i_b}{\mathrm{d}x}\,\mathrm{d}x = \int_{-W}^{W} \left\{ -\frac{F(x)}{\varrho_{sh}}\, v_n(x) + i_n(x) \right\} \mathrm{d}x$$

$$= \int_{-W}^{W} \mathrm{d}x \int_{-W}^{W} \mathrm{d}x'\, F(x)\, G(x,x') \left\{ i_n(x') - \frac{1}{\varrho_{sh}} \frac{\mathrm{d}e_n(x')}{\mathrm{d}x'} \right\} + \int_{-W}^{W} i_n(x)\,\mathrm{d}x$$

$$= \int_{-W}^{W} i_n(x)\,\{1 - v_s(x)\}\,\mathrm{d}x - \int_{-W}^{W} \frac{1}{\varrho_{sh}}\, e_n(x) \frac{\mathrm{d}v_s(x)}{\mathrm{d}x}\,\mathrm{d}x. \qquad (23)$$

In this derivation use has been made of eq. (18) for $v_s(x)$ and the term containing $\mathrm{d}e_n/\mathrm{d}x$ is once partially integrated.

Squaring eq. (23) and averaging with the use of eqs (11), (12) and (13) gives

$$\overline{i_{Bn}^{\,2}} = 2q\,(1-\alpha)\,\Delta f \int_{-W}^{W} J_e(x)\,\{1 - v_s(x)\}^2\,\mathrm{d}x + \frac{4\,kT}{\varrho_{sh}}\,\Delta f \int_{-W}^{W} \left(\frac{\mathrm{d}v_s}{\mathrm{d}x}\right)^2 \mathrm{d}x$$

$$= 2q\,(1-\alpha)\,\Delta f \int_{-W}^{W} J_e(x)\,\{1 - v_s(x)\}^2\,\mathrm{d}x +$$

$$+ \frac{4\,kT}{\varrho_{sh}}\,\Delta f \int_{-W}^{W} F(x)\,v_s(x)\,\{1 - v_s(x)\}\,\mathrm{d}x. \qquad (24)$$

The second term of eq. (24) is written in the final form by partial integration and use of eq. (9).

In the equivalent circuit of fig. 3 noise sources have to be added to represent the noise behaviour. The following noise sources are added (see fig. 4):

(a) A shot-noise current source $i_{n1}$ over the emitter–base junction. The termination of this current source is assumed to be at a point of the base resistance which divides $r_{bb'}$ into a part $p\,r_{bb'}$ at the internal-base-point side and $(1-p)\,r_{bb'}$ at the external-base-point side.

(b) A shot-noise current source $i_{n2}$ over the base–collector junction, terminating at the same point of the base as $i_{n1}$.

(c) A noise voltage source $e_n$ in series with $r_{bb'}$.

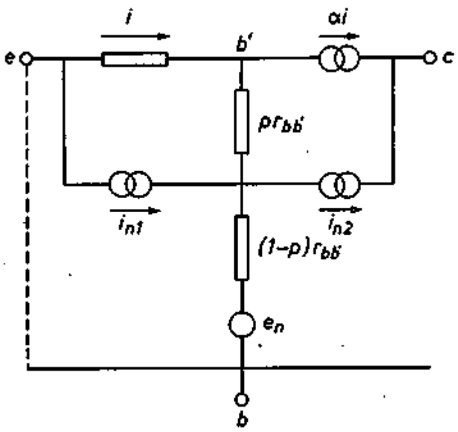


Fig. 4. Small-signal equivalent circuit of the transistor including the noise sources.

The shot-noise current sources are full shot-noise sources of the emitter and collector currents given by [1])

$$\overline{i_{n1}^2} = 2\,q\,I_E\,\Delta f,$$
$$\overline{i_{n2}^2} = 2\,q\,\alpha\,I_E\,\Delta f, \tag{25}$$
$$\overline{i_{n1}\,i_{n2}^*} = 2\,q\,\alpha\,I_E\,\Delta f.$$

The noise voltage source $e_n$ is the thermal-noise voltage of the base resistance $r_{bb'}$ with an effective noise temperature $t_{eff}\,T$:

$$\overline{e_n^2} = 4\,kT\,r_{bb'}\,\Delta f\,t_{eff}. \tag{26}$$

The noise source $e_n$ is uncorrelated with $i_{n1}$ and $i_{n2}$:

$$\overline{e_n\,i_{n1}^*} = \overline{e_n\,i_{n2}^*} = 0. \tag{27}$$

With short-circuited emitter base we easily calculate the base noise current $i_{Bn}$:

$$i_{Bn} = \left\{(i_{n1} - i_{n2})\left(\frac{kT}{q\,I_B} + p\,r_{bb'}\right) - e_n\right\}\left(\frac{kT}{q\,I_B} + r_{bb'}\right)^{-1}.$$

Squaring and averaging gives with the help of eqs (25), (26) and (27):

$$\overline{i_{Bn}^2} = 2\,q\,I_B\,\Delta f\left\{\frac{kT/qI_B + p\,r_{bb'}}{kT/qI_B + r_{bb'}}\right\}^2 + \frac{4\,kT\,r_{bb'}\,\Delta f\,t_{eff}}{(kT/qI_B + r_{bb'})^2}. \tag{28}$$

Equations (24) and (28) have to be identical. Equalizing the shot-noise terms and the thermal-noise-voltage terms separately gives equations for $p$ and $t_{\text{eff}}$:

$$\left(\frac{kT/qI_{\text{B}} + p\, r_{\text{bb'}}}{kT/qI_{\text{B}} + r_{\text{bb'}}}\right)^2 = \frac{\int_{-W}^{W} J_{\text{e}}(x)\,(1 - v_{\text{s}}(x))^2\, \mathrm{d}x}{\int_{-W}^{W} J_{\text{e}}(x)\, \mathrm{d}x}, \tag{29}$$

$$t_{\text{eff}} = \frac{(kT/qI_{\text{B}} + r_{\text{bb'}})^2}{\varrho_{\text{sh}}\, r_{\text{bb'}}} \int_{-W}^{W} F(x)\, v_{\text{s}}(x)\,(1 - v_{\text{s}}(x))\, \mathrm{d}x. \tag{30}$$

## 5. Results of the numerical calculations

Equation (4), which determines the d.c. current crowding due to the potential drop in the base, cannot be solved analytically. Therefore the base region is divided into a number of discrete steps, and the differential equation is replaced by a difference equation. This non-linear two-point boundary-value problem can be solved by the rapid-converging iterative Newton method, described for instance in ref. 6. As soon as we have the solution $V(x)$ of eq. (4) for a certain applied voltage $V_{\text{e}}$ we can calculate the function $F(x)$. With the known $F(x)$ the function $v_{\text{s}}(x)$ can be numerically calculated by replacing the second-order linear differential equation (9) by a difference equation and omitting the noise terms.

With this procedure $V(x)$, $F(x)$, $v_{\text{s}}(x)$ are known functions of $x$ at a number of discrete values of $x$. We are now able to evaluate the various integrals by numerical integration. In this way we calculate $I_{\text{B}}$ (eq. (17)), $r_{\text{bb'}}$ (eq. (21)), $p$ (eq. (29)) and $t_{\text{eff}}$ (eq. (30)). All these calculations are performed and results are given foı a value of the dimensionless constant

$$\frac{q}{kT}\,\varrho_{\text{sh}}\, J_{\text{s}}\, W^2 (1 - \alpha) = 10^{-12},$$

which is a practical value ($\varrho_{\text{sh}} = 5000\ \Omega$, $J_{\text{s}} = 10^{-6}\ \text{A m}^{-2}$, $W = 10^{-5}$ m, $1 - \alpha = 5.10^{-2}$). For each applied $V_{\text{e}}$ the whole procedure has to be repeated.

In fig. 5 the quantity $J_{\text{e}}(x)/J_{\text{e}}(W)$ is given as a function of $x/W$ for various values of $V_{\text{e}}$, which clearly illustrates the increasing current crowding for increasing $V_{\text{e}}$. Figure 6 gives the a.c. voltage $v_{\text{s}}(x)$ in the base as a function of $x/W$ with $V_{\text{e}}$ as parameter.

In fig. 7 $r_{\text{bb'}}$ is given as a function of $V_{\text{e}}$; $r_{\text{bb'}}$ is normalized to $\frac{1}{6}\,\varrho_{\text{sh}}\, W$ which is the well-known value of $r_{\text{bb'}}$ for no current crowding. The dashed curve is the result of Hauser's calculation, who used a number of simplifying assumptions.

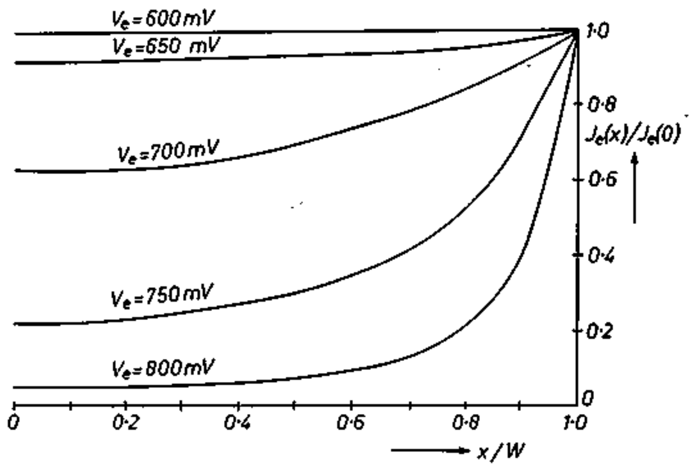


Fig. 5. Emitter-current density as function of $x$ with applied emitter d.c. voltage as parameter.

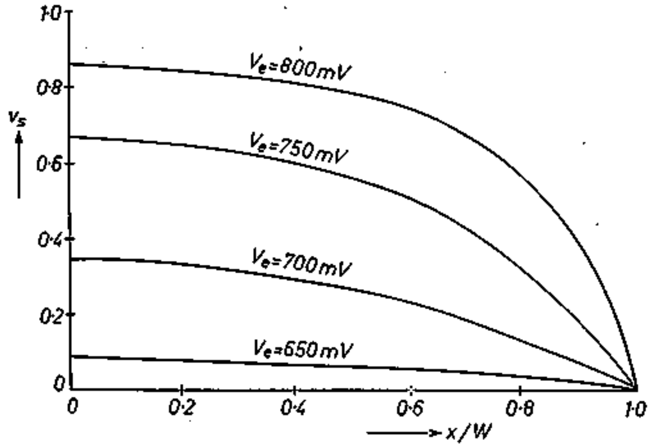


Fig. 6. A.c. potential $v_s(x)$ in the base for an applied emitter a.c. voltage $v_e = 1$, with applied emitter d.c. voltage as parameter.

Figure 8 gives the effective-noise-temperature ratio $t_{eff}$ and the value of $p$, which determines the place where the shot-noise current sources terminate at $r_{bb'}$. When no current crowding occurs we see that $t_{eff} = 1$ and $p = 0$, as is the case in the normally used equivalent circuits.

## 6. Conclusion

The calculations enable us to draw the following conclusions.

With increasing d.c. current crowding the $r_{bb'}$ in our model decreases, which is a well-known effect. The effective noise temperature of the base resistance in the equivalent circuit turns out to decrease with increasing d.c. current crowding. At the same time the shot-noise current sources over the emitter

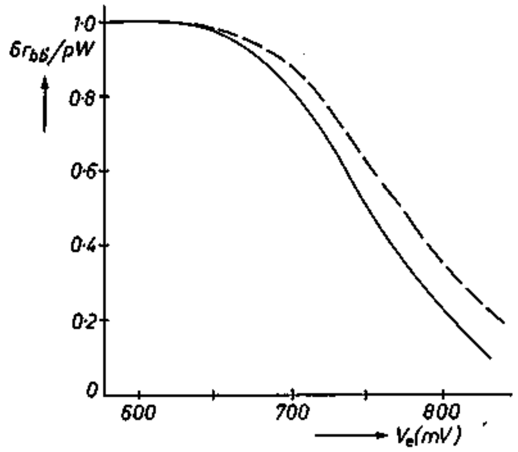


Fig. 7. $r_{bb'}$ as a function of the applied emitter d.c. voltage.

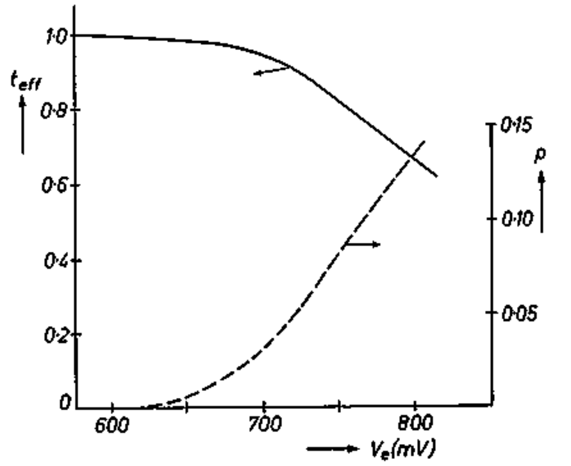


Fig. 8. $t_{eff}$ and $p$ as a function of the applied emitter d.c. voltage.

and collector junctions no longer terminate on the internal base point, but at a point of the resistance $r_{bb'}$ which moves in the direction of b with increasing d.c. current crowding.

For a transistor with appreciable d.c. current crowding it is not permissible to add simply the usual noise sources to the small-signal equivalent circuit used for that transistor. The noise behaviour of a transistor with d.c. current crowding is better than we would expect, if the commonly used noise sources are added to the equivalent circuit. On the other hand, if we want to measure the equivalent base resistance of a transistor from its noise properties, we must take account of the effective noise temperature of the base resistance when d.c. current crowding occurs.

Our calculations were based on a simple model for the transistor, which nevertheless included the essential effects of current crowding. More sophisticated transistor models will also include the effects of decreasing noise temperature of base resistance and changes of termination of the shot-noise current sources, when d.c. current crowding occurs.

*Eindhoven, November 1970*

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